Symmetry, Invariance, and the Circular Geometry of Debt as an explanation of Biblical Sabbatical and Debt laws

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Abstract

I show that the economic activities of buying and selling (the real side of an economy) and lending and borrowing (the financial side) can be represented in a single two dimensional Euclidean geometric space. This model is consistent with the accounting identity approach of Stock Flow Consistent (SFC) macroeconomic models. The central result of this dissertation is that this geometric representation is consistent with the economic outcomes induced by the simultaneous observance of a combination of two biblical laws. These laws are related to the weekly Sabbatical cycle and a seven year debt cycle. This result is in part generated by an original application Pythagoras' theorem to macroeconomics.

JEL-Classification: E191, C0

Keywords: Debt, Credit, Symmetry, Geometry, Sabbath, Stock Flow Consistent, Pythagoras' theorem, Circle

1 Introduction

This dissertation simultaneously takes both an ancient and highly novel approach to macroeconomics. The central research question of this dissertation is how the simultaneous observance of two biblical laws, related to the weekly cycle and a debt cycle, can be captured in a two dimensional geometric representation of the economy. This question is important because it is generally assumed that the analysis of the economy has nothing to do with religion. In contrast, I show the consistency of biblical scripture and a simple geometric approach in macroeconomics. I build the geometry by considering the symmetries implied by accounting identities that must hold in a logically consistent macroeconomic model.

The first addition of this dissertation to the economics literature is that it provides a link between Christian economics and mathematical economics. Secondly, this model provides a framework in which the real and financial sides of the economy are modelled in an integrated way. Finally, it provides an interesting application of Pythagoras' theorem to the macroeconomics.

Two geometric representations of the economy are derived and compared: an economy that does not permit debt and an economy that does permit debt. I derive the ratio of the expected level of debt market activity (lending and borrowing) to goods market activity (buying and selling) by considering the geometric representation of macroeconomic accounting identities. I then show that the ratio of debt market to goods market activity ratio is approximately equal to the same ratio implied by the simultaneous observation of two particular biblical laws. Both the biblical and the geometric approaches imply that, in expectation, the business cycle is six-sevenths the length of the credit cycle. The empirical evidence found in recent literature on the relation between business and financial cycles is somewhat consistent with this result. For instance, recent research has found that financial cycles are generally longer in duration than business cycles.

The rest of the dissertation is broadly structured as follows. In the next section I provide a review of the recent economics literature that has some relation to my dissertation. I then go on to describe the model and its geometric representation. I conclude by showing the equilibrium of the model, which is found by taking expectations, is reflective of the simultaneous observation of two biblical laws.

2 Literature Review

2.1 Biblical Scriptures

Firstly, I present the two biblical laws that are central to this dissertation. In the twentieth chapter of the biblical book of Exodus, the fourth of the ten commandments reads (in verses 8 -11): "Remember the Sabbath day, to keep it holy. Six days you shall labor, and do all your work, but the seventh day is a Sabbath to the Lord your God. On it you shall not do any work, you, or your son, or your daughter, your male servant, or your female servant, or your livestock, or the sojourner who is within your gates. For in six days the Lord made heaven and earth, the sea, and all that is in them, and rested on the seventh day. Therefore the Lord blessed the Sabbath day and made it holy." The Ancient Israelites were here commanded to work for six days a week and then rest on the seventh day of the week. This also means that Israelites were not permitted to work or buy and sell for six-sevenths of their time. Elsewhere in scripture it is made clear that there was to be no buying or selling on this day of rest (Nehemiah, 10:31). There is clear economic content in this commandment as it outlines a seven day repeated cycle of six days of work, and associated buying and selling, and one day of rest per week.

Biblical scripture also dictates a seven year credit cycle. In fifteenth chapter of Deuteronomy it reads (verses 1 and 2): "At the end of every seven years you must cancel debts. This is how it is to be done: Every creditor shall cancel any loan they have made to a fellow Israelite. They shall not require payment from anyone among their own people, because the Lord's time for canceling debts has been proclaimed." This verse dictates that the debt cycle was to occur over seven years: lending and borrowing would occur over seven years but then stop at the end of the seventh year before immediately re-starting.

Concurrently, over this time period, six years of work (and associated buying and selling) will have occurred if the observance of the Sabbath command was simultaneously practised. This implies a ratio of six-sevenths between two activities: that of buying and selling and lending and borrowing. These two laws, if obeyed simultaneously, imply a ratio between the duration of two cycles: the business cycle and the credit cycle.

2.2 Christian Economics

This dissertation further develops themes present in my own paper that was written for the 'Discussion Paper Series' of the Association of Christian Economists (Morgan, 2013). There I consider the relationship between the Sabbath and debt laws. Unlike Morgan (2013), this dissertation abstracts from a particular monetary system, thereby making it more general. However, the Morgan (2013) takes essentially the same approach but with less rigorous justification of the central results.

More generally, there are many papers in the Christian economics literature loosely related to this topic. Those papers mainly relate to what is known as the 'Jubilee year'. A Sabbatical year occurred every seven years, where land was to lie fallow and experience a year of 'rest', whereby it would not be worked. The Jubilee occurred every fiftieth year. At this point, land was to be returned to families who originally owned it. A recent paper related to the Jubilee law is Fanucci (2014), considers the underlying principles of the Jubilee, its wider purpose, and impact upon the society of Ancient Israel. The paper also considers the implications for the modern day global economy in the context of Third World Debt. Other papers in the Christian Economics literature have considered the practical application of the Jubilee laws and have doubted the extent to which they were actually followed in practice or were indeed practicable (Schaefer and Noell, 2005). Anderson (2005) attempts to understand biblical relationships between God, the chosen people of Israel, and the promised land. The concept of Jubilee is considered an important institution by which God provided protection against economic calamity. It was both a year of rest, a sort of super-sabbatical, but also a time when property was to revert to its original owners and debts were forgiven. "People were to return to their family clans and the original land allocations provided for those clans." "Israelites who had been forced to sell their land got it back in the year of Jubilee." The paper goes on to analyse the implications of the law for land ownership and valuation. Mills (2017) considers biblical law as a whole where the production and sale of goods is almost entirely left to market forces, while the laws governing the role of finance are tightly drawn.

2.3 Macroeconomic models of Intertemporal consumption

In models of intertemporal choice a key aspect is the budget constraints of agents. In these models, consumers need not consume all of their income in a given period, which leads to savings (and lending) or borrowing. In overlapping generations models, agents live for two periods and make choices about how much they save and consume. In these models there are two periods and two groups which interact. In the Overlapping Generations model where there is a "pay-as-you-go" social security system that is unfunded, there are direct transfers from the young to the old in the same period. These are forced gifts. These models are relevant to this dissertation as they provide examples of budget constraints that interact. The model in this dissertation also has two groups of agents and two periods. However, the model in this paper analyses the symmetric properties of the budget constraints of two groups: debtors and creditors. In effect, the model in this dissertation presents two groups that act as if they are each country in a two country world within a two-period current account model. I consider, what are, in effect, transactions that occur within the borders of a country and transactions that occur across borders, where the first type of transaction does not affect the current account whilst the second type of transaction does. The symmetry of the budget constraints is akin to the fact that the current accounts of a two country world sum to zero by definition.

2.4 Stock-Flow Consistent macroeconomic models and the methodological approach

Dynamic Stochastic General Equilibrium (DSGE) models have sought to include a financial sector as a friction following the seminal works of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). In general, these financial frictions occur when incomplete markets prevent trade in certain assets. However DSGE models have major drawbacks. For instance, the representative agent approach rules out key macroeconomic interactions by assumption (Stiglitz and Gallegati, 2011). In particular there are no lenders or borrowers, and therefore no credit markets. Stiglitz and Gallegati (2011) argue that financial markets (lending and borrowing) can only be studied within a framework with heterogenous agents. In contrast, DSGE models assume a representative agents for the key sectors of the economy. These representative agents behave optimally in a similar way to an individual firm or consumer. Moreover, DSGE models continue to suffer from the "fallacy of composition" in that they assume that what is true for individual agents is also valid for the aggregated economy (Delli Gatti et al., 2010a). These, and other limitations of DSGE models has led to growing interest in Stock Flow Consistent models (SFC).

SFC models models account for the interactions between all stocks and flows in the economy. The approach is grounded in the pioneering work of Wynne Godley and James Tobin undertaken in the 1970s and 1980s. These models have gained a recent resurgence following the work of Godly and Lavoie (2007). A range of these models have been developed in recent years, including Caverzasi and Godin (2105), Nikolaidi (2014), Brunnermeier, et al (2012). SFC models are characterised by two main components. Firstly, they consist of an accounting approach based on a Quadruple entry system (Copeland, 1949). Copeland studied the 'money flows', and he originated the flow of funds approach. Essentially, this approach leads to a social version of the double-entry accounting system which is doubled to generate a quadruple-entry system. The accounting framework captures the interrelatedness of agent's budget constraints. SFC models integrate in the real and financial sides of the economy by means of a social accounting matrices.

The second key component of SFC models is a set of behavioural equations describing how the system behaves over time. Caiani et al (2016) argue that the most significant advantage of the SFC approach is that it provides a fundamental check of the model logical consistency. The methodological approach I take is closely related to the principles of the accounting framework component of SFC models. This, however, is where my model departs. My model also does not model the financial sector in any detail, effectively abstracting from the sale and purchase of financial assets.

2.5 Credit cycles versus business cycles: empirical approaches

This dissertation considers an ancient approach to credit and business cycles, based on biblical scripture. The current economic literature has found empirical evidence that is consistent with the biblical scripture. For instance, Aikman et al (2015), uses a model used to generate credit cycles which amplify cycles in the real economy relative to a world without credit cycle frictions. Moreover, a range of studies find that the financial cycle is of longer duration than the business cycle, including Drehman (2012), Galati et al (2016) and Herman et al (2015). Schuler et al (2015) find that 'on average financial cycles last 7.2 years and Herman et al (2015) find the business cycle to be 6.3 years in duration. However, these are just two estimates within a large range of empirical values found in the literature. For instance, Gonzalez et al (2015) evaluate credit cycles and business cycles length over several countries using non-parametric de-trending techniques. They find financial cycles averaging about 13 or 14 years, and business cycles 3 to 7.5 years.

3 The model

There are J agents who 'buy and sell' and 'lend and borrow' to one another over two periods of equal length. Each good has a fixed price. Each agent has a credit (debt) balance. An agent's credit balance at the end of each period is the difference between the sum of their incomes and expenditures over that period. An agent is a creditor in the first period if the sum of their first period income exceeds the sum of their first period expenditure. Similarly, a debtor is an agent whose expenditure exceeds their income in a given period. From here on I consider incomes and expenditures in terms of the summation of incomes and expenditures over an entire period.

Any excess income over and above expenditure of a creditor agent is instantaneously translated into loan to a debtor: a type of forced gift. As such, each debtor uses these loaned funds to purchase goods from the creditor group. No aggregate savings are possible and loans are made at zero interest rates. It is assumed that all first period debts are repaid by the end of the second period. This means that each agent in the creditor group in the first period will be in the debtor group in the second period and vice versa. This means that this analysis is about an idealised economy where all debt contracts are fulfilled. Models such as Alvarez and Jermann (2000) and Kehoe and Levine (1993) have endogenous debt limits in the form of individual rationality constraints to ensure that agents cannot enter into a contract in which they would have an incentive to default in some state. This is one method that is used in the existing literature to prevent defaults occurring in a model. I assume that all agents have a finite amount of debt or credit, but that this can be made arbitrarily close to zero.

There are two markets: the debt market (for lending and borrowing) and exchange market (for buying and selling). There are two periods of analysis, with the end of the first period being determined is when the total amount of debt reaches its highest point. This point marks the half-way point in terms of the total amount of joint cumulative output (and income) generated over the two periods. At the start of period one there is no debt in the system. As such, output equals expenditure for each group for the cumulative period up until period one starts. Hence, there is no outstanding debt as every agent has fully funded their expenditure. At the end of period two, cumulative output (and income) of each group of agents, over the two periods, is again equal so that there is no debt at the end of the debt cycle. Over the two periods, by definition, the debt and credit positions of the two representative agents move in equal and opposite directions. More specifically, their shares of total cumulative output must move in equal and opposite directions throughout: debt is a zero sum game

It is a stochastic environment with a probability distribution around each transaction, meaning that the is a random chance that a particular unit of expenditure is assigned to a particular unit of income. This makes sense because each agent repays their debts by the end of the second period. This means that over the whole two periods, both groups have equal incomes.

4 The geometric representation of the model

4.1 Orthogonality and Symmetry

I capture the credit (debt) levels and the accumulated end of period incomes and expenditures of the two groups in two dimensions. This serves to integrate the real and financial sides of the economy. Changes to income and expenditure in this model is represented by a comparative static change of the end of period accumulated incomes and expenditures. This is captured by different examples below.

The x and y axes measure the debtor and creditor group's incomes and expenditures respectively. The positive part of the axis measures income and the negative part measures expenditures. The axes are perpendicular because the respective income and expenditure of each group do not directly feed into the credit balances of the other group. So, the creditor group's incomes and expenditures do not directly feed into the debtor's credit (debt) balance. The aggregate amount of debt in the economy is the sum of the debtor groups income less the sum of their aggregated expenditures.

There are three central symmetries in this model. Firstly, the total amount of income in the economy equals the total amount of expenditure. Secondly, the absolute difference between the creditor group's incomes and expenditures is equal to the aggregate credit (debt) balance. Thirdly, by definition, this is also equal to the absolute difference between the debtor group's income and expenditure.

4.2 The space of potential transactions

A transaction occurs when a unit of income is paired with a unit of expenditure. A central concept I introduce to generate the result is that of a 'space of potential economic transactions'. This is the cartesian product of potential pairings of units of expenditure and income. It is the set of different permutations pairwise matches (tuples) of units of income and expenditure. This is similar to a probability space. I consider the total number of potential pairs of units (or sub-units) or expenditure and income. For example, in the UK, one could label each pound of income and expenditure for each individual. So one can consider all the potential pairings of a pound of expenditure and a pound of income. If there is a re-ordering of the different combinations of buyers and sellers this would mean that the different flows of income and expenditure would be between different buyers and sellers. I keep this space of potential transactions constant when hypothetically comparing different distributions of economic activity shared between debtors and creditor. The different distributions of economic activity represent different levels of debt. This is because the different distributions of economic activity are summarised by different distributions of income and expenditure.

For every $Y \times Y$ space only Y transactions are realisable. So to generate an income level of Y there will be an associated $Y \times Y$ space of potential transactions. There is a simple algorithm to derive an output level when firstly presented with a potential space of transactions. Once one particular transaction is 'selected' from a particular row and column, then that row and column will be 'deleted'. The next transaction is then selected from another row and column. There are Y rows and Y columns, with each potential transaction belonging both to one and only one row and column. This means that in total there are Y transactions that can be realised in an $Y \times Y$ space of potential transactions.

Figure 1 below shows the potential transaction space of 5 units of expenditure and income. In this example $Y = 5$. The first number represents a unit of income and the second number the associated unit of expenditure. One potential realisation is along the diagonal where each labelled unit is matched to its equivalent counterpart.

Figure 1: Cartesian product of Space of potential transactions

5,1	5,2	5,3	5,4	5,5
4,1	4,2	4,3	4,4	4,5
3,1	3,2	3,3	3,4	3,5
2,1	2,2	2,3	2,4	2,5
1,1	1,2	1,3	1,4	1,5

When comparing different distributions of economic activity (incomes and expenditures) distributed between the two groups, the space of potential transactions is kept invariant (constant). This makes sense intuitively as I seek to isolate the geometric characteristics of debt. I want to be able to consider what the economy looks like with differing amounts of debt. However, I cannot simultaneously constrain the level of output to be equal for all differing amounts of debt. This is because this would imply the absence of an intrinsic relationship between debt and income (or output). Either income (and output) levels could be kept invariant when comparing examples or the potential space of economic transactions. However, something must be conserved so to isolate the nature of debt; in a way akin to a controlled experiment. I chose the space of economic transactions as it is a more primitive concept: one must first have a 'potential space' before actual output - made up of a series of transactions - is realised. Any particular realisation of incomes and expenditures (output) is just one realisation of the 'potential space'.

4.3 SFC principles and Within-group and Between-group transactions

The geometric representations described in the next section are consistent with the four main principles of Stock Flow Consistent (SFC) macroeconomic models, which are summarised by Nikoforos and Zezza (2017). The first principle is that of Horizontal consistency: Everything comes from somewhere and goes somewhere: there are no black holes. For instance, income for somebody is a payment from somebody else. This principle is captured by the first symmetry of this model: that total income equals total expenditure. The second principles is that of Vertical consistency: Every transaction involves at least two entries within each unit. For example, a consumer's expenditure implies, say, a reduction in the consumers credit balance. This is captured by the second symmetry of my model. The third principle is that, Every flow implies the change in one or more stocks, for instance, a change in a debt stock. This is captured in my model by conducting comparative statics to consider how changes in income and expenditure levels of each group affect the aggregate stocks of incomes and expenditures in each period. The fourth principle is that of Stocks consistency: The financial liabilities of an agent or sector are the financial assets of some other agent or sector. For example, a loan is a liability for a household and an asset for a bank; a Treasury bond is a liability for the government and an asset for its holder. As a result, the net financial wealth of the system as a whole is zero. This leads to the Quadruple entry accounting system.

Principles two to four a captured because the each axis captures both 'expenditures and incomes' as well as the aggregate debt balance of the economy. In particular, and by definition, the difference between the incomes and expenditures of each group measures the aggregate debt balance of the economy.

In this model there are four types of transaction. Firstly, a creditor agent can spend a unit of money that is paired with unit of income of a another creditor agent. Secondly, a debtor agent can spend a unit of money that is paired with a unit of income for another debtor agent. These first two types are the two examples of within-group transactions. Neither of these transactions will alter the aggregate debt level in an economy as expenditures flow back into the respective group's as income. On the axes of a two dimensional geometric representation of the model, this is be represented by an increase (decrease) in the positive part of the axis associated with an equal increase (decrease) of the negative part. These changes cancel out. This leaves the absolute difference between the two, the aggregate debt (credit) balance, unchanged.

The third and fourth types of transaction are between-group transactions. The first of these is when a creditor agent spends a unit of money that is paired with a unit of income for a debtor agent. This type of transaction will decrease the overall level of debt. This is because it increases the credit balance of the debtor group whilst simultaneously decreasing the credit balance of the creditor group. The second type of between-group transaction is when a debtor agents spends a unit of money that is paired with a unit of income for a creditor agent. This type of transaction will increase the overall level of debt.

In summary, within-group transactions are the associated with an economy where debt tends to zero in the limit. It tends to, but does not equal zero, so that both groups remain defined. So within-group transactions can be thought of as describing an economy which has no debt instruments: a no-debt economy. In contrast, between-group transactions do alter the aggregate debt level. Therefore, they can be associated with an economy that does have debt instruments.

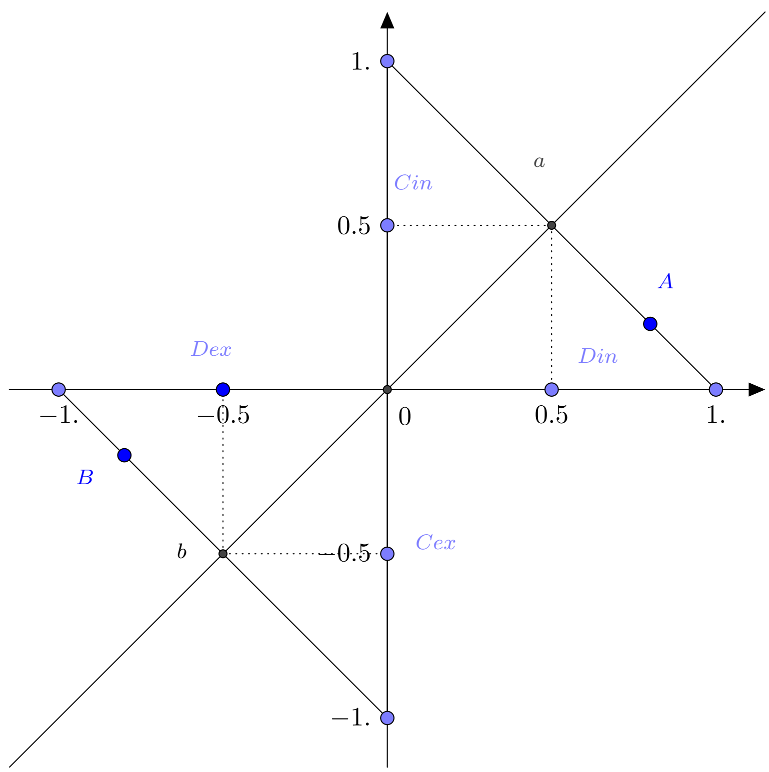
I assume that there is an equal likelihood of a unit of expenditure flowing within-groups as opposed to between-groups. This is an 'independence assumption' with regard to the market for incomes and expenditures (the goods market) and the market for lending and borrowing (the financial market). I here assume that when any arbitrarily chosen agent decides to make a purchase or sale, the debt (credit) balance of the counterpart seller or buyer does not entire into their decision process; it is not included in their preferences.

4.4 No-debt economy: within-group transactions

I firstly describe the geometric representation of an economy with not debt. I will then compare this to an economy which permits debt.

Figure 2, as all figures in this dissertation, only the first of two periods is captured. In each case, a second period debt repayment outcome can be found by a reflection of the geometric representation in the line $y = -x$.

Figure 2: No-debt Economy



In an extreme case the creditor group accounts for all the potential within-group income. I normalise this total amount of income and expenditure to each be the unit. This is the baseline normalisation to compare different alternative hypothetical realisations of the aggregate flows of incomes and expenditures.

Figure 2, represents this by two points: $(0, 1)$, the creditor group's income, and $(0, -1)$, the creditor group's aggregate expenditure. This shows the creditor group's expenditure is one unit and income also one unit. In contrast, the debtor group's income and expenditure level will be zero. Likewise, the other polar example is where the debtor group accounts for 100 percent of the within-group activity. This is represented by points $(1, 0)$, representing the debtor group's income and point $(-1, 0)$ representing the debtor group's expenditure.

The total joint 'space' is constrained to be a one-dimensional length of one unit. This can be 'distributed' between the two groups so as to leave the total lengths (incomes) equal to the unit. This length (one-dimensional space) that is kept invariant along the lines $y+x = 1$ for incomes and $y+x = -1$ for expenditures. To find a particular combination of incomes an arbitrary point along the line $y = 1 - x$ is selected. This point is then projected onto onto the axes. The projection onto the y-axis gives the creditor group's normalised income level and the projection onto the x-axis gives the debtor group's normalised income level. The means that the potential space of transactions will belong to a one-dimensional space. This is because the units of income associated with units of expenditure will always be found along the same axes, as these must belong to the same group. However, the total amount of within-group transactions will be distributed between two one-dimensional spaces: the two axes.

I assume that at any randomly chosen point in time, the two groups have an equal chance of receiving an arbitrary unit of income. This means that the expected distribution of total incomes, when all transactions are within-groups, is at point $(0.5, 0.5)$. Similarly, the expected distribution of expenditures will be at point $(-0.5, -0.5)$.

4.5 Debt-economy: between-group transactions

The baseline case I use for between-group transactions is again when the creditor group receives 100 percent of the income that flows between the two groups in total. This allows it to be comparable to the within-group scenario. It means that both the no-debt (within-group) and debt (between-group) representations have a unit of creditor group income as the baseline for comparing alternative distributions of economic activity. In this between-group baseline case there is a key difference in the geometry when compared to the within-group example. In the between-group example, 100 percent of the expenditure must come from the debtor group.

With between-group transactions, two dimensions are required in the extreme cases. To represent the space of potential transactions there are two square spaces of potential pairwise matches of income and expenditure. The first square, $S1$ is represented in the second quadrant of the axes. It has with sides described by the positive part of the y-axis (the creditor group's income) and the negative part of the x-axis (the debtor group's expenditure). This occurs because, for solely between-group transactions, the creditor group's income must be associated with the debtor group's expenditure. The area of this first square represents the cartesian product of labelled units of the debtor group's income with the creditor group's expenditure. The second square, $S2$ represents the cartesian product of the creditor group's expenditure and the debtor group's income. This is depicted in the fourth quadrant of the unit circle. It has sides characterised by the positive part of the x-axis (the debtor group's income) and the negative part of the y-axis (the creditor group's expenditure). This occurs because, for solely between-group transactions, the debtor group's income must be associated with the creditor group's expenditure. This space is kept invariant when considering different distributions of economic activity with and between the debtor and creditor groups.

In summary, if the cartesian product representing the space of potential transactions is for within-group transactions, this space must be represented in one-dimension. In contrast, if the pairing involves units of income flowing from one group and going to another as expenditure (between-group transactions) then the space of potential transactions must be represented in two dimensions.

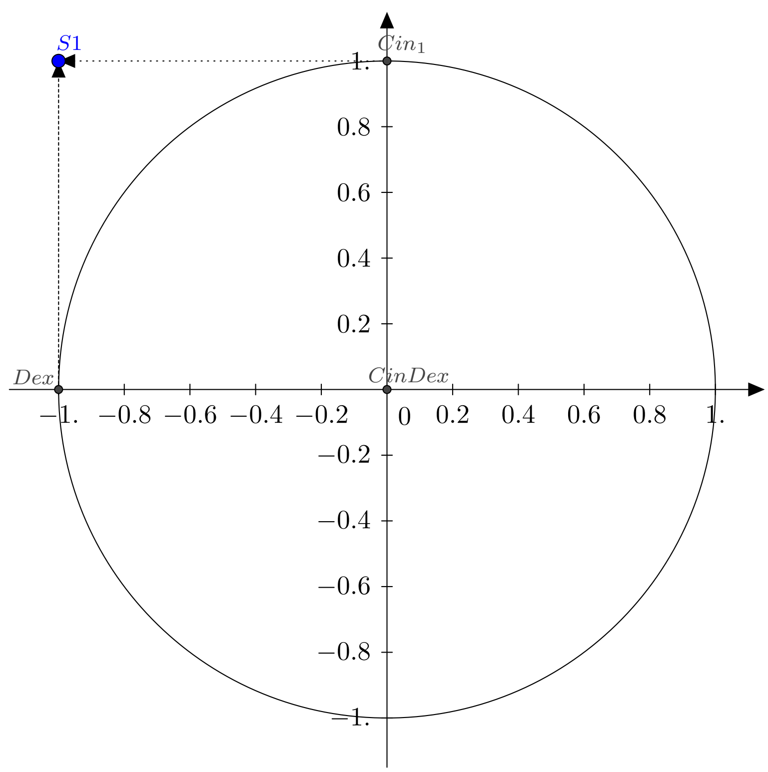
4.6 Examples of between-group geometric representations

Below I move between three examples (of an infinite number of potential examples) of between-group distributions of economic activity. When I do this, I constrain the total aggregated absolute space of the two squares, $S1$ and $S2$ to be a constant total the squared unit. This leaves the space of potential transactions invariant.

4.6.1 Geometric representation variant 1: Maximum Debt

As the level of debt reaches a maximum I obtain figure 3 below.

Figure 3: Maximum Debt-economy



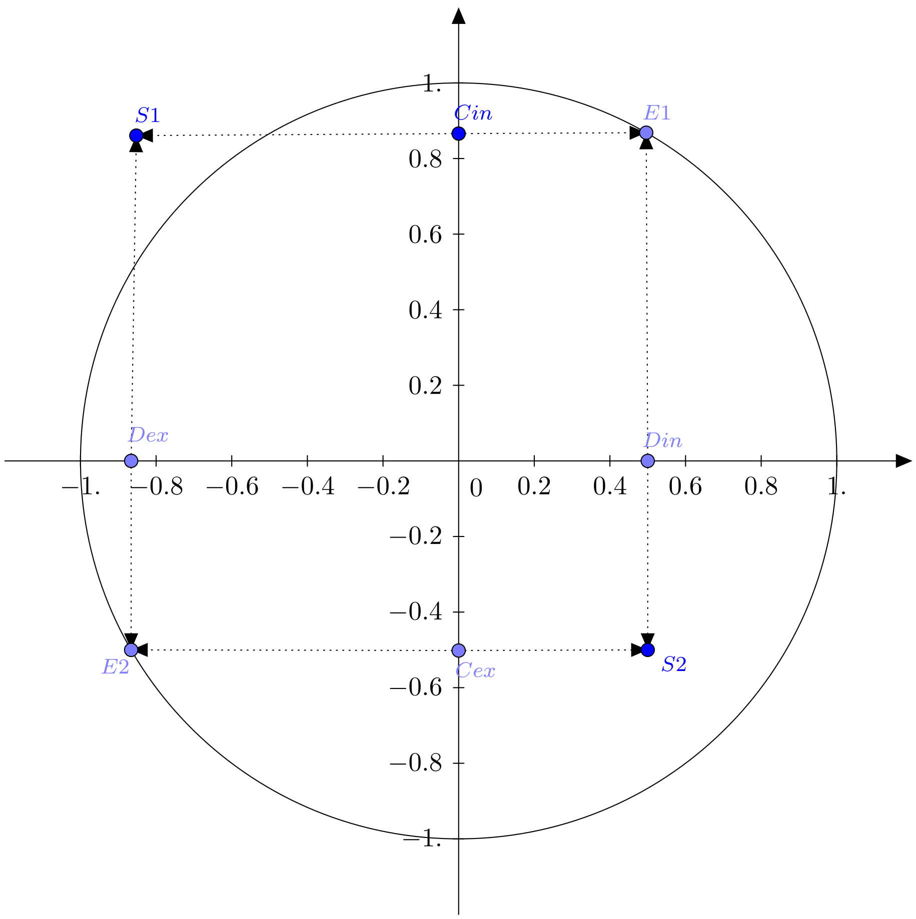
This shows a case where the creditor group has zero expenditure and the debtor group have zero income. With 'maximum debt', the creditor group contributes one hundred percent of output and receives 100 percent of the income. The absolute size of the respective credit balances are equal as is demanded by the symmetry implied by the definition of aggregate credit balances. The co-ordinates of the income level of the creditor group is $(0, 1)$, whilst symmetry implies that the expenditure level of the debt. In contrast, by definition, the credit balance of the debtor group is the negative unit. In each case, by symmetry, the credit balances of each group will be equal in absolute value but of opposite signs.

The space of potential transactions is the cartesian product of all sub-units of income and expenditure. This space is a square space with sides determined by Cin (the creditor group's income level) and Dex (the debtor group's expenditure level), which captures the symmetry between creditor expenditure and debtor expenditure for transactions that occur only between the creditor and debtor groups. This space is equal to the squared unit, which is of course a two dimensional space.

4.6.2 Geometric representation variant 2: Intermediate case

Figure 4 below shows an example of an intermediate case. In this case the respective income level of each group are unequal. The creditor group is receiving more income than the debtor group. The symmetry that holds is that the absolute difference between the incomes and expenditures of each group is at all times equal. So Cin (creditor group income) equals Dex (debtor group expenditure) by symmetry, and this provides a square space of potential economic transactions, $S1$. Similarly, Din (debtor group income) is equal in absolute length to Cex (creditor groups expenditure). This is because all of the creditor group's expenditure goes to the debtor group for between-group transactions, defining a square of potential transaction $S2$.

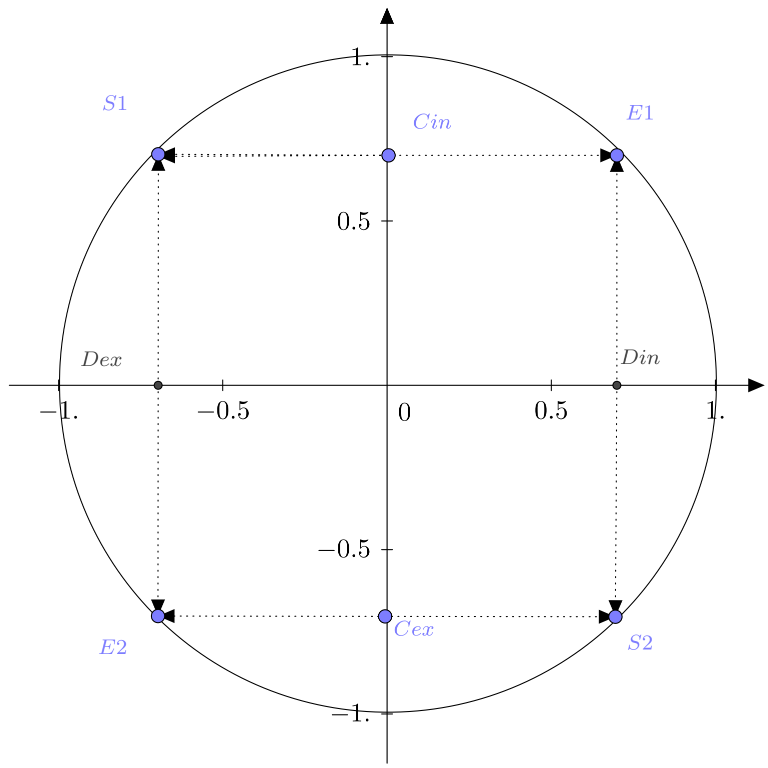
Figure 4: Intermediate Debt Economy



4.6.3 Geometric variant 3: Zero Debt-economy

Figure 5 below shows the limiting case of the economy as debt tends to zero.

Figure 5: Minimum debt



The debtor group's income (Din) must equal creditor group's expenditure (Cex). As such, there is a square, $S2$, where the point $S2$ is the intersection of the where the projections of the creditor group's expenditure (Cex) value, along the x-axis, and the debtor group's income value (Din) value, along the y-axis, meet. This is once again a space of potential mappings from expenditures to incomes.

The absolute length of each is $\frac{1}{\sqrt{2}}$. Likewise the square $S1$ is formed as the debtor group's expenditure (Dex) must equal the creditor group's income (Cin). The projection of the debtor group's income level (along the y-axis) and the projection of the creditor groups income level (along the x-axis) meet on the unit circle at point $E1$. By symmetry, the equivalent point with respect to expenditures occurs at point $E2$.

In the example of minimum debt, both incomes are represented by the point $\frac{1}{\sqrt{2}}$, which approximately equals 0.7. This is the level because when $\frac{1}{\sqrt{2}}$ is squared one has $\frac{1}{2}$. The sum of two identical squares of size $\frac{1}{\sqrt{2}}$ is the square unit. As such, the size of the space of potential transactions is equal to the example when the creditor received the unit as 100 percent of income (Figure 3). Hence, when lending and borrowing are permitted, the between-group flows economy has an expected income level of approximately 0.7 for each group. This is because, in a debt-economy, incomes are expected to be equal at any arbitrarily given point of time over two periods. The reason for this is that is has been assumed that all debts are repaid by the end of the second period. As such, both incomes (and expenditures) will be equal at the end of the period, and so are expected to be equal at any arbitrarily chosen point along the way.

For debt related economic activity (between-group transactions), points E1 and E2, in all potential distributions of income, lie on the unit circle, but only in the first and third quadrants of the unit circle. This is where where both x and y are simultaneously positive or negative.

5 Between-group transactions and Pythagoras' theorem

Between-group transactions relate to the unit circle. In an economy with debt, which is associated with between-group transactions, the debtor group must have expenditure exceeding income. In contrast, the creditor group must have income exceeding their expenditure. Moreover, the expenditure of the debtor group equals the income of the creditor group. For between-group transactions, the positive part of the x-axis represents the debtor group's first period income. By symmetry, this is equal to the creditor group's first period expenditure, which is measured along the negative part of the y-axis. These form the sides one one square. Similarly, the negative part of the x-axis measures the debtor group's expenditure, which is equal to the creditor group's income, which is measured along the negative part of the y-axis. These two squares together represent the total 'space of economic transactions'. I normalise the total 'space of economic transactions' to be the unit, so that the total joint area of the two squares in total sum to the squared unit. As the total joint area of the two squares are restricted to the unit square; Pythagoras' theorem holds: $x^2 + y^2 = 1$. However, this only holds in the first and third quadrant of the circle.

This Pythagoras' theorem result requires that the total area of the space of potential pairings of incomes and expenditure remains invariant between examples. The rationale for doing this is that it enables the isolation of the geometric properties of debt. This in some sense keeps the 'space' of the economy the same. Unless the combined area remains constant when considering a debt economy in comparison to a non-debt economy, then we would be considering debt and economic growth contemporaneously, rather than isolating the mechanics of debt. Each of the two squares can be viewed as different sized 'non-debt sub-economies', where incomes equal expenditures.

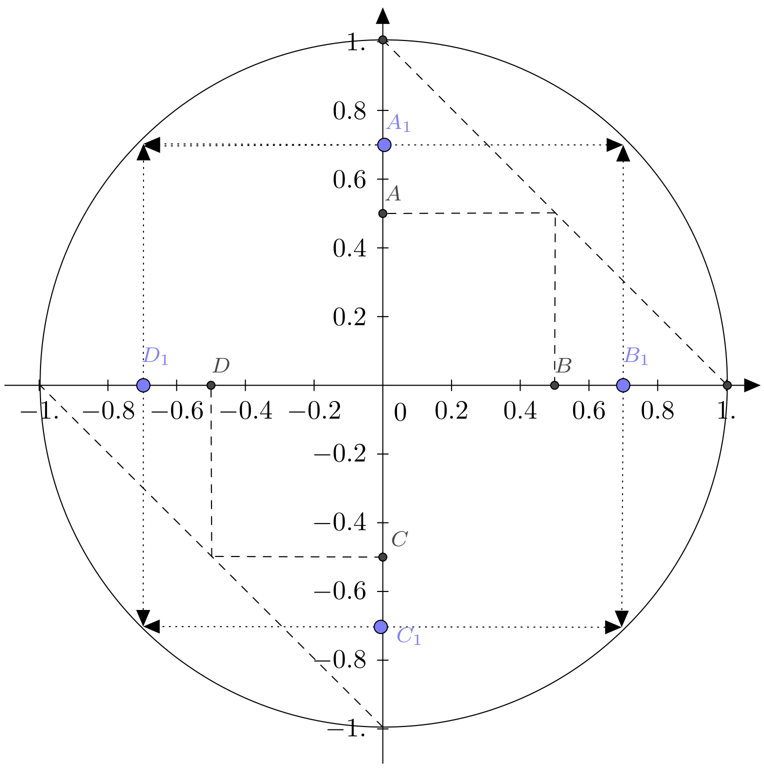
There are three central factors that generate the Pythagoras' theorem result. Firstly, the credit balances of the two groups meet orthogonally. Secondly, there is a symmetry that holds. This is that the extent to which the creditor group's first period income exceeds their expenditure is identical to the absolute measure of the extent to which the debtor group's first period expenditure exceeds their income.

6 Relation to biblical laws

I now relate the results outlined above to the biblical laws considered. The biblical laws dictate that the temporal space of credit cycle is seven years. Meanwhile, the Sabbath commandment stipulates one day of rest out of seven each week. When these laws are observed simultaneously this means that over seven years of a credit cycle the there is six years of buying sand selling in total. This leads to a ratio of goods market and debt market activity, which is six-sevenths.

The no-debt (within-group transaction) geometry has an expected income level of both groups at point $(0.5, 0.5)$ points A and B in Figure 6 below.

Figure 6: Expected outcomes



In contrast the debt (between-group) output equilibrium is satisfied at point $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$, points A_1 and B_1 in figure 6 below. The intuition for why the expected output and income realisations of an economy with debt exceeds that of an economy without debt, is related to the equations of the circle: $y^2 + x^2 = 1$, in comparison to that of the straight line of $y + x = 1$. These two constraints, respectively, outline the constraints, or rather trade-offs, that depict the no-debt and debt economies. The debt economy constraint of the circle formula, $y^2 + x^2 = 1$, is a weaker constraint than that of the non-debt economy, $y + x = 1$.

There is an economic intuition to this difference in expected income levels in the case where no debt is permitted when contrasted to the case when debt is permitted. Consider all the potential realisations of output levels of the debtor and creditor groups within a specified range. If no debt is permitted, for all these realisations of output to translate to economic transactions occurring between the groups, the income levels of both groups cannot diverge from their respective expenditure levels, and must also be equal in aggregate to each other. In contrast, when borrowing and lending is available (and hence the full range between-group transaction) there are further possibilities. With debt, income and expenditure levels can diverge as long as the amount that the creditor group's income exceeds its expenditure is equal and opposite to the amount the the debtor group's expenditure exceeds its income. In the case of a no-debt economy, none of these additional potential transaction opportunities are realisable. Instead, there could only be transactions up to, but not exceeding, the output level of the debtor group in each period.

Figure 6 shows the expected outcomes of the within-group (no-debt transactions) and the between-group (debt transactions) in an integrated geometric representation. Expected one period debt related activity is at about 0.7 ($\frac{1}{\sqrt{2}}$) and non-debt activity at about 0.5. Both lie on the 45 degree line as, in expectation, the income levels of both groups are equal at any arbitrarily chosen point in time over the two periods. This assumes that the expectation is not conditioned on knowing which group is in creditor or debt in which period.

However, debt related, between-group economic activity is also associated with buying and selling, but just in two as opposed to one dimension. There is an equal likelihood of within-group or between-group activity by the 'independence assumption'. As such, on average, buying and selling requires a space that is the average of 0.5 and $\frac{1}{\sqrt{2}}$, which is approximately 0.6. However, pure debt-related (between-group) activity requires an expected space of 0.7. This yields a ratio between the two of approximately six-sevenths. This space can be thought of as time, as time is required to facilitate economic activity. Hence, if each unit of space is considered to be a year, a debt-economy requires seven years within which to operate compared to a non-debt economy that requires six years in expectation.

7 Conclusions

This dissertation develops a geometric representation of a macroeconomic model of debt. This geometric representation has symmetry and Pythagoras' Theorem at its core. The equilibrium of this model is obtained by taking expectations of an economy which has random expenditures when conditioned only on the flows of income and expenditure within and between debtor and creditor groups. The expected income outcomes of the two groups of this economy yield are ratio between real (buying and selling) and financial (lending and borrowing) economic activity of approximately six-sevenths. This is consistent with the ratio generated by the simultaneous observance of two laws: the Sabbath commandment and the law dictating a seven year credit cycle, which is six-sevenths.

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